

Super-Maxwell Actions with Manifest Duality

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Superstring field theory was recently used to derive a four-dimensional Maxwell action with manifest duality. This action is related to the McClain-Wu-Yu Hamiltonian and can be locally coupled to electric and magnetic sources.

In this letter, the manifestly dual Maxwell action is supersymmetrized using $N=1$ and $N=2$ superspace. The $N=2$ version may be useful for studying Seiberg-Witten duality.

1. Introduction

The study of N=2 super-Yang-Mills theory has recently become popular due to the duality conjecture of Seiberg and Witten.[1] This duality conjecture asserts that by exchanging electrically and magnetically charged states, the N=2 quantum theory at strong and weak coupling can be related.

This quantum duality conjecture is related to a continuous classical symmetry of N=2 super-Maxwell theory which rotates the electric field into the magnetic field and vice versa. In the standard super-Maxwell action, classical duality symmetry is not manifest since it rotates Bianchi identities into equations of motion. This is related to the fact that magnetic sources cannot couple locally in the standard action.

Recently, superstring field theory was used to derive a ten-dimensional action for a self-dual five-form field strength.[2] After dimensional reduction, this gave a manifestly dual action for a four-dimensional Maxwell field. The action contains an infinite number of fields and can be obtained from the McClain-Wu-Yu Hamiltonian[3] by performing a Legendre transformation.[4] Since duality is manifest, it is easy to couple locally to electric and magnetic sources.[5]

In this letter, the manifestly dual Maxwell action is generalized to super-Maxwell actions with N=1 or N=2 four-dimensional supersymmetry. These actions can couple locally to electric and magnetic sources and are written in N=1 or N=2 superspace. The N=2 version may be useful for studying Seiberg-Witten duality since, at the classical level, duality is now manifest.

In section II of this letter, the manifestly dual Maxwell action is reviewed. In section III, the action is supersymmetrized in N=1 superspace, and in section IV, in N=2 superspace. In section V, some conclusions are presented.

2. Review of the manifestly dual Maxwell action

In references [2] and [5], superstring field theory in the presence of D-branes was used to obtain a ten-dimensional action for a self-dual five-form field strength in the presence of sources. After dimensional reduction to four dimensions, this gave a manifestly dual Maxwell action in the presence of electric and magnetic sources. This action is constructed from a complex vector field, $E_p = A_p + iB_p$, and an infinite set of real anti-symmetric tensor

fields, $F_{(n)}^{pq}$, where (n) runs from 0 to ∞ . In the presence of a fermionic dyon source with electric charge e and magnetic charge g , the action is

$$\begin{aligned} \mathcal{S} = & \int d^4x [-F_{(0)}^{pq}(\partial_p A_q - \frac{1}{2}\epsilon_{pqrs}\partial^r B^s) \\ & + \frac{1}{2}F_{(1)}^{pq}(\partial_p A_q + \frac{1}{2}\epsilon_{pqrs}\partial^r B^s) - \frac{1}{2}\sum_{n=0}^{\infty}(F_{(2n)}^{pq} + F_{(2n+2)}^{pq})F_{(2n+1)pq} \\ & + \bar{\psi}^{\dot{\alpha}}(i\partial_p + eA_p + gB_p)\sigma_{\alpha\dot{\alpha}}^p\psi^\alpha] \end{aligned} \quad (2.1)$$

where α and $\dot{\alpha}$ are two-component Weyl spinor indices, σ^0 is the identity matrix, and σ^i are the Pauli matrices for $i = 1$ to 3. Note that (2.1) can also be written as

$$\begin{aligned} \mathcal{S} = & \int d^4x [-\frac{1}{4}(F_{(0)}^{\alpha\beta}\partial_{\alpha\dot{\alpha}}\bar{E}_{\dot{\beta}}^{\dot{\alpha}} + \bar{F}_{(0)}^{\dot{\alpha}\dot{\beta}}\partial_{\alpha\dot{\alpha}}E_{\dot{\beta}}^{\alpha}) \\ & + \frac{1}{8}(F_{(1)}^{\alpha\beta}\partial_{\alpha\dot{\alpha}}E_{\dot{\beta}}^{\dot{\alpha}} + \bar{F}_{(1)}^{\dot{\alpha}\dot{\beta}}\partial_{\alpha\dot{\alpha}}\bar{E}_{\dot{\beta}}^{\alpha}) \\ & - \frac{1}{4}\sum_{n=0}^{\infty}((F_{(2n)}^{\alpha\beta} + F_{(2n+2)}^{\alpha\beta})F_{(2n+1)\alpha\beta} + (\bar{F}_{(2n)}^{\dot{\alpha}\dot{\beta}} + \bar{F}_{(2n+2)}^{\dot{\alpha}\dot{\beta}})\bar{F}_{(2n+1)\dot{\alpha}\dot{\beta}}) \\ & + \bar{\psi}^{\dot{\alpha}}(i\partial_{\alpha\dot{\alpha}} + \frac{1}{2}(e - ig)E_{\alpha\dot{\alpha}} + \frac{1}{2}(e + ig)\bar{E}_{\dot{\alpha}\alpha})\psi^\alpha] \end{aligned} \quad (2.2)$$

where $F_{(n)}^{\alpha\beta} = \frac{1}{2}\sigma_p^{\alpha\dot{\alpha}}\bar{\sigma}_{q\dot{\alpha}}^\beta F_{(n)}^{pq}$, $E_{\alpha\dot{\alpha}} = (A_p + iB_p)\sigma_{\alpha\dot{\alpha}}^p$, $\bar{\sigma}^{p\alpha\dot{\alpha}} = \epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}\sigma_{\beta\dot{\beta}}^p$, and $\partial_{\alpha\dot{\alpha}} = \sigma_{\alpha\dot{\alpha}}^p\partial_p$.

The above action is manifestly invariant under the continuous duality rotation

$$F_{(2n)}^{\alpha\beta} \rightarrow e^{i\kappa} F_{(2n)}^{\alpha\beta}, \quad F_{(2n+1)}^{\alpha\beta} \rightarrow e^{-i\kappa} F_{(2n+1)}^{\alpha\beta}, \quad (2.3)$$

$$E^p \rightarrow e^{i\kappa} E^p, \quad (e + ig) \rightarrow e^{i\kappa}(e + ig)$$

where κ is a real constant. It is also invariant under the local gauge transformation

$$E_p \rightarrow E_p + 2\partial_p \Lambda, \quad \psi^\alpha \rightarrow e^{(ie+g)\Lambda + (ie-g)\bar{\Lambda}} \psi^\alpha. \quad (2.4)$$

Since an infinite number of fields are present in (2.1), there are subtleties involved in analyzing solutions to the equations of motion. To remove these subtleties, only solutions with a finite number of non-zero fields will be allowed. In other words, at each point in spacetime, only a finite number of on-shell fields will be allowed to be non-zero. This

restriction is similar to that of reference [4] and guarantees that the energy is finite and well-defined.¹

The equations of motion from varying $F_{(n)}^{pq}$, A_p , B_p , and ψ^α are easily calculated to be

$$\begin{aligned} F_{(0)}^{pq} - \frac{1}{2}(\partial^p A^q - \partial^q A^p + \epsilon^{pqrs} \partial_r B_s) &= -F_{(2)}^{pq} = F_{(4)}^{pq} = -F_{(6)}^{pq} = \dots, \\ \partial^p A^q - \partial^q A^p - \epsilon^{pqrs} \partial_r B_s &= F_{(1)}^{pq} = -F_{(3)}^{pq} = F_{(5)}^{pq} = \dots, \\ \partial_q F_{(0)}^{pq} &= e\bar{\psi}^{\dot{\alpha}} \sigma_{\alpha\dot{\alpha}}^p \psi^\alpha, \quad \frac{1}{2}\epsilon_{pqrs} \partial^q F_{(0)}^{rs} = g\bar{\psi}^{\dot{\alpha}} \sigma_{\alpha\dot{\alpha}}^p \psi^\alpha. \\ (\partial_p - ieA_p - igB_p) \sigma_{\alpha\dot{\alpha}}^p \psi^\alpha &= 0. \end{aligned} \tag{2.5}$$

Solutions to (2.5) with a finite number of non-zero fields satisfy

$$\begin{aligned} F_{(0)}^{pq} = \partial^p A^q - \partial^q A^p = \epsilon^{pqrs} \partial_r B_s, \quad F_{(n+1)}^{pq} = F_{(n+2)}^{pq} &= 0, \\ \partial_q F_{(0)}^{pq} = e\bar{\psi}^{\dot{\alpha}} \sigma_{\alpha\dot{\alpha}}^p \psi^\alpha, \quad \frac{1}{2}\epsilon_{pqrs} \partial^q F_{(0)}^{rs} &= g\bar{\psi}^{\dot{\alpha}} \sigma_{\alpha\dot{\alpha}}^p \psi^\alpha. \\ (\partial_p - ieA_p - igB_p) \sigma_{\alpha\dot{\alpha}}^p \psi^\alpha &= 0. \end{aligned} \tag{2.6}$$

These are the standard Maxwell equations in the presence of electric and magnetic sources.

3. Manifestly dual N=1 super-Maxwell action

To generalize (2.2) to N=1 superspace, one introduces a complex scalar superfield, V and \bar{V} , and an infinite set of chiral spinor superfields, $W_{(n)}^\alpha$ and $\bar{W}_{(n)}^{\dot{\alpha}}$, satisfying $\bar{D}_{\dot{\alpha}} W_{(n)}^\beta = D_\alpha \bar{W}_{(n)}^{\dot{\beta}} = 0$. ($D_\alpha = \partial/\partial\theta^\alpha + i\bar{\theta}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}}$ and $\bar{D}_{\dot{\alpha}} = \partial/\partial\bar{\theta}^{\dot{\alpha}} + i\theta^\alpha \partial_{\alpha\dot{\alpha}}$ are the usual N=1 fermionic derivatives.) This is the N=1 analog of the fields in (2.1) since the standard N=1 super-Maxwell action uses a real scalar superfield whose field strength is a chiral spinor superfield.[6]

The N=1 super-Maxwell action in the presence of a dyonic Wess-Zumino scalar multiplet[7] is

$$\begin{aligned} \mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta} &[-\frac{1}{8}(W_{(0)}^\alpha D_\alpha \bar{V} + \bar{W}_{(0)\dot{\alpha}} \bar{D}^{\dot{\alpha}} V) \\ &+ \frac{1}{16}(W_{(1)}^\alpha D_\alpha V + \bar{W}_{(1)\dot{\alpha}} \bar{D}^{\dot{\alpha}} \bar{V})] \end{aligned} \tag{3.1}$$

¹ In reference [4], the fields with label (n) were restricted to satisfy $|F_{(n)}| < 1/n$ for $n > N$. This type of restriction is inappropriate for fermionic fields.

$$\begin{aligned}
& -\frac{1}{4} \int d^4x d^2\theta \sum_{n=0}^{\infty} (W_{(2n)}^\alpha + W_{(2n+2)}^\alpha) W_{(2n+1)\alpha} \\
& -\frac{1}{4} \int d^4x d^2\bar{\theta} \sum_{n=0}^{\infty} (\bar{W}_{(2n)\dot{\alpha}} + \bar{W}_{(2n+2)\dot{\alpha}}) \bar{W}_{(2n+1)}^{\dot{\alpha}} \\
& + \frac{1}{2} \int d^4x d^2\theta d^2\bar{\theta} \bar{\Phi} e^{(e-ig)V+(e+ig)\bar{V}} \Phi
\end{aligned}$$

where Φ is a chiral scalar superfield satisfying $\bar{D}^{\dot{\alpha}}\Phi = 0$.

This action is manifestly invariant under the duality rotation

$$W_{(2n)}^\alpha \rightarrow e^{i\kappa} W_{(2n)}^\alpha, \quad W_{(2n+1)}^\alpha \rightarrow e^{-i\kappa} W_{(2n+1)}^\alpha, \quad (3.2)$$

$$V \rightarrow e^{i\kappa} V, \quad (e+ig) \rightarrow e^{i\kappa}(e+ig),$$

and under the gauge transformation

$$V \rightarrow V + (D)^2\Lambda + (\bar{D})^2\bar{\Lambda}, \quad \Phi \rightarrow e^{(ig-e)(\bar{D})^2\bar{\Lambda}-(ig+e)(\bar{D})^2\bar{\Lambda}}\Phi, \quad (3.3)$$

where $(D)^2 = \frac{1}{2}D^\alpha D_\alpha$.

Assuming that only a finite number of on-shell fields are non-zero, the equations of motion for (3.1) are

$$W_{(0)}^\alpha = \frac{1}{4}(\bar{D})^2 D^\alpha (V + \bar{V}) = \frac{1}{4}(\bar{D})^2 D^\alpha (V - \bar{V}), \quad W_{(n+1)}^\alpha = W_{(n+2)}^\alpha = 0, \quad (3.4)$$

$$\frac{1}{4}D_\alpha W_{(0)}^\alpha = (e+ig)\bar{\Phi} e^{(e-ig)V+(e+ig)\bar{V}} \Phi,$$

$$(D)^2(e^{(e-ig)V+(e+ig)\bar{V}}\Phi) = 0.$$

These equations are easily seen to be the N=1 generalization of (2.6) where $F_{(n)}^{\alpha\beta} = \frac{i}{2}(D^\alpha W_{(n)}^\beta + D^\beta W_{(n)}^\alpha)|_{\theta=\bar{\theta}=0}$, $E_{\alpha\dot{\alpha}} = D_\alpha \bar{D}_{\dot{\alpha}} V|_{\theta=\bar{\theta}=0}$, and $\psi_\alpha = D_\alpha \Phi|_{\theta=\bar{\theta}=0}$.

4. Manifestly dual N=2 super-Maxwell action

To generalize (2.2) to N=2 superspace, one introduces a complex SU(2) triplet of superfields, V_{jk} and \bar{V}_{jk} , and an infinite set of chiral scalar superfields, $W_{(n)}$ and $\bar{W}_{(n)}$, satisfying $\bar{D}_\alpha^j W_{(n)} = D_\alpha^j \bar{W}_{(n)} = 0$. ($D_\alpha^j = \partial/\partial\theta_j^\alpha + i\bar{\theta}^{j\dot{\alpha}}\partial_{\alpha\dot{\alpha}}$ and $\bar{D}_\alpha^j = \partial/\partial\bar{\theta}_j^{\dot{\alpha}} + i\theta^{j\alpha}\partial_{\alpha\dot{\alpha}}$ are the usual N=2 fermionic derivatives where j is an SU(2) doublet index.) This is the

N=2 analog of the fields in (2.1) since the standard N=2 super-Maxwell action uses a real triplet of superfields whose field strength is a chiral scalar superfield.[8]

The N=2 super-Maxwell action in the presence of a dyonic Fayet-Sohnius scalar hypermultiplet[9] is

$$\begin{aligned}
\mathcal{S} = & \int d^4x d^4\theta d^4\bar{\theta} \left[-\frac{1}{48} (W_{(0)} D^{j\alpha} D_{\alpha}^k \bar{V}_{jk} + \bar{W}_{(0)} \bar{D}_{\dot{\alpha}}^j \bar{D}^{k\dot{\alpha}} V_{jk}) \right. \\
& + \frac{1}{96} (W_{(1)} D^{j\alpha} D_{\alpha}^k V_{jk} + \bar{W}_{(1)} \bar{D}_{\dot{\alpha}}^j \bar{D}^{k\dot{\alpha}} \bar{V}_{jk}) \left. \right] \\
& - \frac{1}{4} \int d^4x d^4\theta \sum_{n=0}^{\infty} (W_{(2n)} + W_{(2n+2)}) W_{(2n+1)} \\
& - \frac{1}{4} \int d^4x d^4\bar{\theta} \sum_{n=0}^{\infty} (\bar{W}_{(2n)} + \bar{W}_{(2n+2)}) \bar{W}_{(2n+1)} \\
& + \frac{1}{2} \int d^4x d^2\theta^+ d^2\bar{\theta}^+ \int du (\bar{\Phi}^+)^* (D^{++} + (e - ig)V^{++} + (e + ig)(\bar{V}^{++})^*) \Phi^+
\end{aligned} \tag{4.1}$$

where $V^{++} = (D^+)^2 (\bar{D}^+)^2 u_j^- u_k^- V^{jk}$, $(\bar{V}^{++})^* = (D^+)^2 (\bar{D}^+)^2 u_j^- u_k^- \bar{V}^{jk}$, $D_{\alpha}^{\pm} = u_j^{\pm} D_{\alpha}^j$, $\bar{D}_{\dot{\alpha}}^{\pm} = u_j^{\pm} \bar{D}_{\dot{\alpha}}^j$, $D^{++} = u_j^+ \partial / \partial u_j^-$, and Φ^+ is an analytic superfield satisfying $D_{\alpha}^+ \Phi^+ = \bar{D}_{\dot{\alpha}}^+ \Phi^+ = 0$.

As discussed in reference [10], u_j^{\pm} are harmonic variables which are needed for coupling a hypermultiplet in N=2 superspace. They are complex variables satisfying $u_j^+ u^{j+} = u_j^- u^{j-} = 0$ and $u_j^+ u^{j-} = 1$. The bar operation acts on all fields as complex conjugation, while the $*$ operation acts only on the u_j^{\pm} variables as $(u_j^{\pm})^* = \pm u_j^{\mp}$. Note that $(\bar{u}^{j+})^* = u_j^+$, so $D_{\alpha}^+ (\bar{\Phi}^+)^* = \bar{D}_{\dot{\alpha}}^+ (\bar{\Phi}^+)^* = 0$. Therefore, one only needs to integrate the source term over $d^2\theta d^2\bar{\theta}$ since it is annihilated by D_{α}^+ and $\bar{D}_{\dot{\alpha}}^+$. Integration over the u_j^{\pm} variables is defined by $\int du 1 = 1$ and $\int du u_{(j_1}^+ \dots u_{j_M}^+ u_{k_1}^- \dots u_{k_N}^- = 0$. For more details on harmonic superspace notation, see reference [10].

The N=2 action of (4.1) is manifestly invariant under the duality rotation

$$W_{(2n)} \rightarrow e^{i\kappa} W_{(2n)}, \quad W_{(2n+1)} \rightarrow e^{-i\kappa} W_{(2n+1)}, \tag{4.2}$$

$$V_{jk} \rightarrow e^{i\kappa} V_{jk}, \quad (e + ig) \rightarrow e^{i\kappa} (e + ig),$$

and under the gauge transformation

$$V_{jk} \rightarrow V_{jk} + 3D^{l\alpha} \Lambda_{jkl\alpha} + 3\bar{D}_{\dot{\alpha}}^l \bar{\Omega}_{jkl}^{\dot{\alpha}},$$

$$\Phi^+ \rightarrow e^{(D^+)^2(\bar{D}^+)^2[(ig-e)f_{++++}-(ig+e)(\overline{f_{++++}})^*]}\Phi^+, \quad (4.3)$$

where Λ_{jkl}^α and $\Omega_{jkl}^{\dot{\alpha}}$ are complex SU(2) quadruplets, $f_{++++} = u_+^j u_+^k u_+^l u_+^m (D_j^\alpha \Lambda_{klm\alpha} + \bar{D}_{j\dot{\alpha}} \bar{\Omega}_{klm}^{\dot{\alpha}})$, and $(\overline{f_{++++}})^* = u_+^j u_+^k u_+^l u_+^m (\bar{D}_{j\dot{\alpha}} \bar{\Lambda}_{klm}^{\dot{\alpha}} + D_j^\alpha \Omega_{klm\alpha})$.

Assuming that only a finite number of on-shell fields are non-zero, the equations of motion for (4.1) are

$$W_{(0)} = \frac{1}{24}(\bar{D})^4 D^{j\alpha} D_\alpha^k (V_{jk} + \bar{V}_{jk}) = \frac{1}{24}(\bar{D})^4 D^{j\alpha} D_\alpha^k (V_{jk} - \bar{V}_{jk}), \quad (4.4)$$

$$W_{(n+1)} = W_{(n+2)} = 0,$$

$$\begin{aligned} \frac{1}{24} D^{j\alpha} D_\alpha^k W_{(0)} &= (e + ig) \int du (\overline{\Phi^+})^* u_+^j u_+^k \Phi^+, \\ (D^{++} + (e - ig)V^{++} + (e + ig)(\overline{V^{++}})^*) \Phi^+ &= 0. \end{aligned}$$

These equations are easily seen to be the N=2 generalization of (2.6) where $F_{(n)}^{\alpha\beta} = \frac{1}{2} D_j^\alpha D^{j\beta} W_{(n)}|_{\theta=\bar{\theta}=0}$, $E^{\alpha\dot{\alpha}} = \frac{1}{36} (D_j^\alpha D^{j\beta} D_{l\beta}) (\bar{D}_k^{\dot{\alpha}} \bar{D}^k_{\dot{\beta}} \bar{D}_m^{\dot{\beta}}) V^{lm}|_{\theta=\bar{\theta}=0}$, and $\psi^\alpha = \int du D_+^\alpha \Phi^+|_{\theta=\bar{\theta}=0}$.

5. Conclusions

In this letter, N=1 and N=2 super-Maxwell actions were constructed with manifest duality. Since these actions can be coupled locally to electric and magnetic sources, the N=2 version may be useful for studying Seiberg-Witten duality.

The original form for the manifestly dual Maxwell action was found by computing the contribution of massless Ramond-Ramond fields in the closed superstring field theory action.[2] Since this computation used the Ramond-Neveu-Schwarz formalism of the superstring (including some extra non-minimal fields[11]), the result was not manifestly spacetime-supersymmetric. For describing four-dimensional compactifications of the Type II superstring, there also exists a manifestly N=2 super-Poincaré invariant formalism.[12][13] It would be interesting to try to directly derive the N=2 super-Maxwell action of (4.1) using superstring field theory in the super-Poincaré invariant formalism. This may be possible since, unlike the standard N=2 super-Maxwell action, (4.1) does not require restricted chiral superfields satisfying $D^{j\alpha} D_\alpha^k W = \bar{D}_\alpha^j \bar{D}^{k\dot{\alpha}} \bar{W}$. As was discussed in [13], restricted chiral superfields are unnatural in the super-Poincaré invariant formalism since there is no two-dimensional analog of restricted chirality.

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